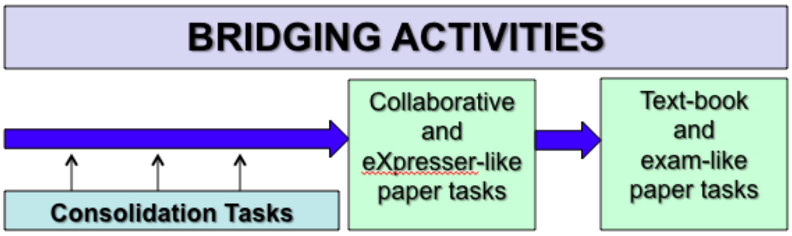
BRIDGING ACTIVITIES

These activities are designed to *assess* and *consolidate* learning that has taken place during the activities using eXpresser, and to support the students in making links between the eXpresser work and the algebra they will meet as part of their 'normal' paper-based curriculum.

Some activities are short, open-ended questions and challenges where students are required to apply what they have learnt with eXpresser to pattern-based, or figural sequences. In these questions, the focus is on how students analyse and describe the structures they perceive in figural sequences (rather than algebraic manipulation). Such questions are useful to assist the teacher in assessing the learning that has taken place during the earlier eXpresser activities and in drawing out students' conceptions and constructions of how patterns and their rules might be described.

We have designed four types of bridging activities, which are shown below:



The schematic presentation of the Bridging Activities

(i) consolidation tasks; short tasks that are used to encourage students to reflect back on their interactions with eXpresser,

(ii) collaborative tasks in which students are asked to decide and then justify if different algebraic rules are equivalent (or not) (see for example one presented on page 50 as an extension task),

(iii) 'eXpresser-like' paper tasks, which are figural pattern generalisation tasks as they had seen on the computer but presented on paper so without the dynamic aspects and links, and

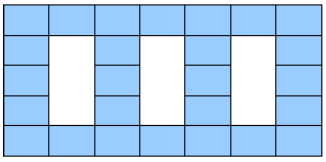
(iv) text-book or examination tasks.

Activity 1 – Traintracks on paper (Consolidation task)

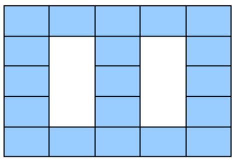
Title	Traintracks on Paper
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constant in the model • Express relations between variables algebraically • Reason mathematically by conjecturing relationships and generalisations • Freely move between different numerical, algebraic and diagrammatic representations • Interpret mathematical relationships algebraically
Teacher Notes	• This activity is a useful consolidation exercise, by way of extension or homework, after doing the Traintracks activity on eXpresser. The aim is for the focus on examining structure when building the model on the eXpresser to be carried over and used when looking at patterns on paper.

Task / Activity

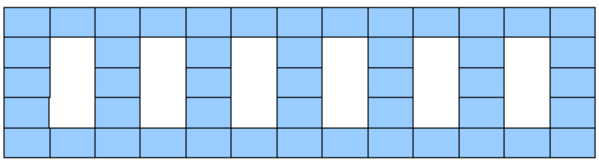
Here are three pictures of the Traintracks on Paper model. Use these, and your work on eXpresser, to answer the questions. In your answers, show how you got to your answer, and show as much working as you can.



Model 3



Model 2



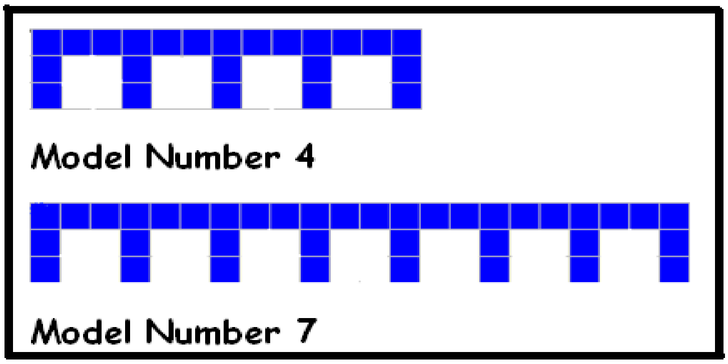
Model 6

1) How many blue tiles would be needed to make Model 12?
2) How many blue tiles would be needed to make Model 50?
3) How many blue tiles would be needed to make Model 1?
4) How many blue tiles would be needed to make Model 200?
5) If we use ‘M’ to stand for the model number, how many blue tiles would be needed to make Model ‘M’?
6) Use the space below to explain the different parts of your rule – use the diagrams provided or draw your own if it helps

Activity 2 – Bridges (eXpresser paper-based task)

Title	Bridges
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constant in the model • Express relations between variables algebraically • Use/Apply a general rule Generate terms of a sequence that are not sequential
Teacher Notes	Variations of this question are common in assessments and offer students the chance to apply what they learned from their interactions with eXpresser on a task, which involves an eXpresser-like model, and teachers the chance to assess students' structural thinking and also identify possible linear scaling issues. For example, students are expected to use their derived rule to answer questions 2, 3 and 4 below. However, our own and our teacher collaborators' experiences has revealed that students often resort to linear scaling and double their answer for Model Number 5 to find the answer for Model Number 10 (Q3) or multiply by 10 the answer for Model Number 10 to find Model Number 100 (Q4). It might also be useful to use this activity to start looking at the numerical sequence that is generated when the total number of tiles is calculated for each model number. This model lends itself to an initial investigation of the numerical representation of the sequence, as the number of blue tiles corresponds to the term, or model number, which makes the arithmetic sequence easier to see.

Task / Activity



1) Find the rule for the number of tiles for any Model Number
2) Find the number of tiles for Model Number 5
3) Find the number of tiles for Model Number 10
4) Find the number of tiles for Model Number 100

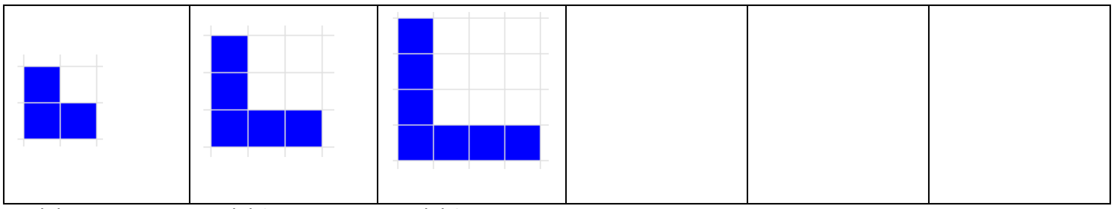
Activity 3 – L-Shapes (eXpresser paper-based task)

Title	L-Shapes
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constant in the model • Express relations between variables algebraically • Simplify and manipulate algebraic expressions to maintain equivalence • Reason mathematically by conjecturing relationships and generalisations and developing an argument using mathematical language
Teacher Notes	This activity consolidates work on eXpresser involving creating expressions and finding rules. It is designed as a classroom activity to be done in groups, so that students can discuss and justify their decisions about how the written rules fit the images and the algebraic expressions, but it can be done with students working individually, in which case provide questioning/ discussion time. Formal algebraic language is introduced, but it is backed up by the construction-based language of eXpresser, such as “growing” building blocks, tiles, model numbers, etc. Prior knowledge of algebraic notation is not necessary, although if students are not familiar with such things as 2L meaning 2xL then the last exercise may best be done in a class plenary. The FENCES activity, which follows, develops the rule-finding theme of this activity, but is more challenging; it is more abstract and differs from the eXpresser concept of using the model number for the variable. Encourage students to try to build the model on eXpresser as a homework activity – it will challenge thinking about the placement of patterns (as the default placement is downwards, whereas the pattern grows upwards), as well as how to link patterns in a model using one unlocked number.

<u>Task / Activity</u>

The aim of this activity is to start describing patterns by using algebra. Patterns can often be **seen** quite clearly in diagrams, and **described in words**. Learning how to describe a pattern **using algebra** is the next step and is an important mathematical skill. It gives us another way to see similarities and differences.

Here are the first three shapes in the L pattern. Draw the next 3 models in the pattern in the empty boxes.



Model 1 Model 2 Model 3

How many tiles high would the 10th model be? ______________________

How many tiles would there be in the 10th model? ____________________

Write a sentence describing how the pattern grows:

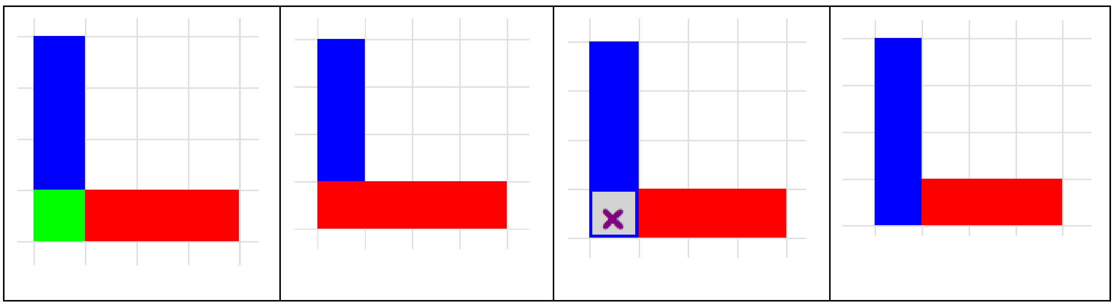
Here are some other students' rules for the pattern. Read the rules and look at the pictures of model number 3 of the pattern below. Write the name of the student below the picture that describes that student's description.

Anna's rule
"There are two patterns, one on the left and one on the right. The left hand side pattern has one more tile than the model number.
The right hand side pattern has the same number of tiles as the model number."

Bertie's rule
"There are two patterns, one on top of the other.
The top pattern goes up and has the same number of tiles as the model number.
The bottom pattern goes out to the right and has one more tile than the model number."

Cyril's rule
"There is a building block of just one tile and then there are two patterns.
Both patterns have the same number of tiles as the model number. One grows upwards and one grows outwards."

David's rule
"There are two patterns, one growing up and one growing out.
Both patterns have one more tile than the model number.
The patterns overlap on the first tile, so for each model number you have to take away one."



Different configuration for Model Number 3

We can use these rules to find the **total** number of tiles used for different model numbers. For example, using Anna's rule, the 16^{th} model will have 16 tiles in the right hand side pattern and 17 in the left hand side pattern. 17 + 16 = 33.

Use **Bertie's** rule to find the number of tiles in the 19^{th} model:
Use **Cyril's** rule to find the number of tiles in the 12^{th} model:

Use **David's** rule to find the number of tiles in the 25^{th} model:
We can write the rule for the total number of tiles using **algebra**:

If “L” is the model number then, using Bertie’s rule, the number of tiles for any model number can be written as:

$$L \quad + \quad L + 1$$

To find the number of tiles for the 30th model, we **substitute** 30 for L in the rule:

$$30 \quad + \quad 30 + 1 \qquad = \qquad 61$$

Use substitution to find the number of tiles in the 18th model number:

Peter has found the number of tiles in his model number to be 53. What model number did he substitute for “L”?

Look at the rules below. Write the name of the other students below their rule:

L + L + 1	2(L +1) - 1	1 + 2L	L+1 + L
BERTIE			

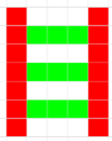
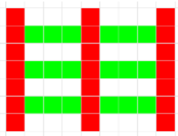
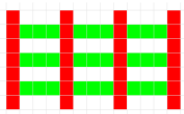
Extension: Rearrange the terms in the rules above to show that each rule can be written the same way.

Activity 4 – Fences

Title	Fences
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constants in the model • Express relations between variables algebraically • Simplify and manipulate algebraic expressions to maintain equivalence • Reason mathematically by conjecturing relationships and generalisations and developing an argument using mathematical language
Teacher Notes	This activity follows on from L-Shaped, and works on developing rules and expressions. The eXpresser metaphor for operating on variables is linking patterns, as nearly all the models in the package are created with just one variable (or “unlocked number” (the exception is the extension activity GRID). The activity HELP TIM guides students through the linking patterns and expressing the repetition of one pattern in terms of another. FENCES focuses on this idea of dependence, and develops expressing one variable in terms of another. Make it clear to students that the “rule” or expression they are finding in this exercise is not a model rule in the eXpresser sense, but a rule connecting the number of posts and the number of rails. (An extension or homework task may be to find the model rule for the total number of tiles in the model.) The focus, however, of this activity is to take the “expression-making” skill that the eXpresser develops in finding model rules beyond finding nth term rules in sequences, and gives a way to connect variables. Substituting different values helps see how the rule works, and is invaluable in getting students to test out their rules. Have students give their rule to others for others, or to the class to test as a group by substituting different values.

<u>Task / Activity</u>

Here are the first three models of the Fence pattern. Look at the number of posts (red) and rails (green). Fill in the table.

Model	Number of posts (*p*)	Number of rails (*r*)
		
		
		

Here are three rules that connect the number of rails and the number of posts in the fences:

Rule A
To find the number of rails, take the number of posts, then subtract 1, and multiply the answer by 3

Rule B
To find the number of rails, take the number of posts, and multiply by 2, then subtract 3 and add the number of posts

Rule C
To find the number of rails, take the number of posts, then multiply by 3 and subtract 3

If we let r stand for the number of rails and p stand for the number of posts, then we can write Rule A in algebra as:

$$\boldsymbol{r = 3p - 3}$$

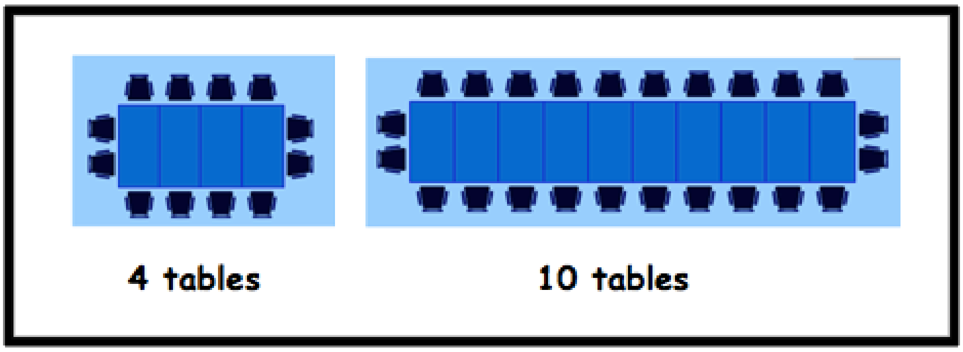
Write the other rules using algebra:

Rule B: $\boldsymbol{r =}$

Rule C: $\boldsymbol{r =}$

Activity 5 – Tables and Chairs

Title	Tables and Chairs
Mathematical Objectives	• Solve a figural pattern generalisation task and find a general rule • Identify the variables and the constant in the model • Express relations between variables algebraically • Use/Apply a general rule
Teacher Notes	Tasks of this type could be given as a homework or final assessment to identify whether students can solve figural pattern generalisation tasks in general.



1) Find the general rule for the number of chairs for any number of tables
2) Use your rule to find the number of chairs for 20 tables
3) Use your rule to find the number of chairs for 200 tables
4) If I have 26 chairs, how many tables do I need?